Knot contact homology

· Very strong invariant of knots via contact topology and homological algebra.

symplectic manifold: Manifold with non-degenerate closed 2-form.
· A form relates differentials of curves on a manifold
· This 2 form is called a "symplectic form"
· Suffices to study even-dimensional manifolds

Example: $\left(\mathbb{R}^{2n}, \sum_{i=1}^{n} dx_i \wedge dx_{2i}\right)$ is a symplectic manifold.

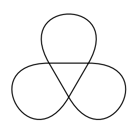
contact manifold: A $(2n+1)$-dimensional manifold M with a contact form (or contact structure).

· A contact form is a 1-form ω on M such that $\omega \wedge (d\omega)^{\wedge n} \neq 0$ everywhere on M
· Frobenius (subbundle of TM is integrable iff it comes from a regular foliation) says nowhere integrable
· The contact structure is $\xi = \ker(\omega)$, a "transversally orientable hyperplane field"

Example: $\left(\mathbb{R}^{2n+1}, dx_{2n+1} - \sum_{i=1}^{n} x_{2i} dx_i\right)$ is a contact manifold.
· Darboux: with change of coordinates, any contact form locally looks like this, on any contact M

Legendrian submanifold: If (M, ξ) is a contact manifold and $L \subset M$ is an integral submanifold (submanifold where $\int_{T_p L} \omega = 0$ for all $p \in L$) of ξ with $\dim(L) = n$, then L is called a Legendrian submanifold.

Example: Let $n = 1$, $(\mathbb{R}^3, dz - ydx)$ be the contact manifold. It is possible to choose L such that the projection of L onto the xy-plane is a knot diagram.



$*$ Knot contact homology tries to answer the question when $L, L' \subset M$ Legendrian submanifolds are isotopic through Legendrian submanifolds.

differential graded algebra: A graded algebra A (direct sum of with a map $d : A \to A$ such that $d^2 = 0$ and follows the Leibniz rule $(d(ab) = (da)b + (-1)^{\deg(a)}(db))$.

Legendrian contact homology of L: A differential graded algebra defined as