The homotopy category

Jānis Lazovskis

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Lurie's "Higher Topos Theory," Chapter 1.2.3
Cisinski's "Higher Categories and Homotopical Algebra," Chapter 1.6

1 Overview and setup

Overview from adjunctions. 1-cats and ssets, induced by scats and ssets.

$\mathfrak{C}$ defined explicitly $\to N'$ is its right adjoint $\to$ induces Ho-N adjunction.

$\text{Obj}(\pi(C)) = C_0$. Morphisms are harder.

2 Construction of morphisms

Define $\text{Edge}(x, y)$
Define homotopic edges
Prop: homotopy is equiv rel on $\text{Edge}(x, y)$
Define $\text{Hom}_{\pi(\mathcal{C})}(x, y)$ as set. composition is well-defined.
Check identity and associativity to have that $\pi(\mathcal{C})$ is category

3 Examples

Eg1. View $\mathcal{C} \in \text{Cat}_1$ as an ∞-cat. Then $\text{Ho}(\mathcal{C}) = \mathcal{C}$ because $\mathcal{C}_2 = \mathcal{C}_1$.
Eg2. Let X be a top space. $\text{Sing}(X)_n := \{\gamma \colon |\Delta^n| \to X \text{ continuous}\}$. Then $\text{Ho}(\text{Sing}(X)) = \Pi_1(X)$.