1. Oscillatory Motion

$$F = -kx$$
$$x(t) = A\sin(\omega t - \phi)$$
$$v(t) = A\omega\cos(\omega t - \phi)$$
$$a(t) = -A\omega^2\sin(\omega t - \phi)$$
$$v_t = \omega r$$
$$a_t = r\alpha$$
$$a_r = \omega^2 r$$
$$E_k = \frac{1}{2}I\omega^2$$
$$E_k = \frac{1}{2}kA^2\sin^2(\omega t)$$
$$E_p = \frac{1}{2}kA^2\cos^2(\omega t)$$
$$F_f = -bv$$
$$x(t) = Ae^{-bt/2m}\cos(\widetilde{\omega}t + \phi)$$
$$\widetilde{\omega} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$
$$F_d(t) = F_\circ\cos(\omega t)$$
$$A(\omega_d) = \frac{F_\circ}{m\sqrt{\left(\omega_d^2 - \omega_\circ^2\right)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{L}} = \sqrt{\frac{mgd}{I}} = \sqrt{\frac{\kappa}{I}} = \frac{2\pi}{T}$$
$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{I}{mgd}}$$

F	: Force, N
k	: Elastic modulus
x	: Displacement, m
v	: Velocity, ms^{-1}
a	: Acceleration, ms^{-2}
t	: Time, s
A, r	: Amplitude (radius) of oscillation, m
ω	: Angular frequency, s^{-1}
ϕ	: Phase shift, rads
T	: Period, s
m	: Mass, kg
L	: Length of pendulum, m
v_t	: Tangential velocity, ms^{-1}
a_t	: Tangential acceleration, ms^{-2}
a_r	: Radial acceleration, ms^{-2}
α	: Angular acceleration, ms^{-2}
E_k	: Kinetic energy, J
E_p	: Potential energy, J
I	: Moment of inertia
d	: Distance from pivot to center of mass, m
F_f	: Damping force, N
b	: Constant of proportionality, kgs^{-1}
$\widetilde{\omega}$	: Varying angular frequency of system, s^{-1}
F_d	: Driving force, N
$F_\circ$	: Initial force, N
$\omega_\circ$	: Natural frequency
ω_d	: Driving frequency

2. Waves

$$
\begin{aligned}
y(x,t) &= f(x - v_w t) \\
y(x,t) &= A\cos(\kappa x - \omega t - \phi) \\
v_w &= \frac{\lambda}{T} = \frac{\omega}{\kappa} = \sqrt{\frac{F_t}{\mu}} \\
P &= \frac{1}{2}\mu\omega^2 A^2 v_w \\
dE_k &= \lim_{\Delta x \to 0} \Delta E_k \\
E_k(\lambda) &= \int_0^{\lambda} dE_k \\
&= \frac{A^2\omega^2\mu\pi}{2k}
\end{aligned}
$$

y	: Displacement of medium from equilibrium, m
$f(x)$	: Function of wave pulse at any point in time
κ	: Wave number
v_w	: Wave speed, ms^{-1}
λ	: Wavelength, m
F_t	: Force of tension, N
μ	: Linear density, kgm^{-1}
P	: Power derived from wave, Js^{-1}
E_k	: Kinetic energy of the wave, J

3. Sound waves

$$
\begin{aligned}
s(x,t) &= s_{max}\cos(\kappa x - \omega t) \\
B &= \frac{-V\Delta\mathbb{P}}{\Delta V} \\
v_w &= \sqrt{\frac{\gamma\mathbb{P}}{\rho}} \\
v_w &= \sqrt{\frac{\text{ϡ} k_B T}{m}} \\
\Delta\mathbb{P}(x,t) &= \Delta\mathbb{P}_{max}\sin(\kappa x - \omega t) \\
\Delta\mathbb{P}_{max} &= \rho v_s \omega s_{max} \\
P &= \frac{1}{2}\rho A v_w (\omega s_{max})^2 \\
I &= \frac{P}{A} = \frac{P}{4\pi r^2} \\
f' &= f\left(\frac{v_o}{v_o \pm v_s}\right) \quad \cdot\text{moving source} \;\; \cdot\text{no change in } \lambda \\
f' &= f\left(\frac{v_o \pm v_s}{v_o}\right) \quad \cdot\text{moving observer} \;\; \cdot\text{change in } \lambda
\end{aligned}
$$

s	: Displacement from equilibrium, m
B	: Bulk modulus, Pa
V	: Volume, m^3
$\mathbb{P}$	: Sound pressure, Pa
γ	: Gas characteristic constant Monatomic gases : $^5/_3$ Diatomic gases : $^7/_5$
ρ	: Density of medium, kgm^{-3}
ϡ	: Adiabatic index (isentropic expansion factor)
A	: Cross-sectional area of path of wave, m^2
I	: Wave intensity, $Js^{-1}m^{-2}$
f	: Frequency, s^{-1}
f'	: Doppler effect frequency, s^{-1} $+$: Source receding / observer approaching $-$: Source approaching / observer receding
v_s	: Speed of source, ms^{-1}
v_o	: Speed of observer, ms^{-1}

4. Superposition

superposition principle : If two or more travelling waves are moving through a medium the resulting wave function is the sum of two individual wave functions. Holds for small oscillations.

dispersion : The phenomenon that wave speed depends on frequency.

$$y(x,t) = 2A\sin(\kappa x + \bar{\phi})\sin(\omega t + \frac{1}{2}\Delta\phi_{12})$$
$$\bar{\phi} = \frac{\phi_1 + \phi_2}{2}$$
$$\Delta\phi_{12} = \phi_2 - \phi_1$$
$$L = \frac{m\lambda}{2}$$
$$\Delta\phi = \kappa\Delta L$$

y	: Equation of motion for interfering waves
ϕ_1, ϕ_2	: Phase shifts of two interfering waves
L	: Length of string, m
m	: Mode number, $m \in \mathbb{N}$
$\Delta\phi$	: Phase difference of two interfering waves
ΔL	: Path difference of two interfering waves

5. Interference

light : In a vacuum, consists of oscillating electric and magnetic fields, such that the directions of the two different oscillations are perpendicular to each other.

Huygens' principle : Each point on a wavefront acts as a disturbance of the medium in which it is propagating and thus is a source for further waves.

diffraction : Ability of waves to travel around corners.

monochromatic : Of a single wavelength. White light is polychromatic.

coherent : Having a constant phase relationship.

** Number of fringes corresponds to number of wavelengths by which two paths differ.

$$\frac{\lambda_1}{\lambda_2} = \frac{v_{w1}}{v_{w2}} = \frac{\sin(\theta_1)}{\sin(\theta_2)}$$
$$n_i = \frac{c}{v_{wi}}$$
$$n_1\sin(\theta_1) = n_2\sin(\theta_2)$$
$$\Delta\phi = \kappa n\ell$$
$$\Delta L = d\sin(\theta) = m\lambda\ ,\ \ m \in \mathbb{N}$$
$$\ell_{OPL} = n\ell$$

λ	: Wavelength of interfering waves
v_w	: Wave speed of interfering waves
θ	: Angle from normal of incident and refracted waves
n_i	: Index of refraction of material i
c	: Speed of light in a vacuum, ms^{-1}
v_{wi}	: Speed of wave in material i, ms^{-1}
$\Delta\phi$	: Phase shift of wave
ℓ	: Length traveled by wave in medium, m
ΔL	: Path difference of wave
d	: Distance between two slits, m
ℓ_{OPL}	: Optical path length

6. Quantum Mechanics

$$E = hf = \hbar\omega$$

$$\hbar = \frac{h}{2\pi}$$

$$q_e V_s = hf - \Phi$$

$$\Phi = hf_\circ$$

$$p = \frac{h}{\lambda}$$

$$\frac{\hbar}{2} \leqslant \Delta x \Delta p$$

$$\leqslant \Delta E \Delta t$$

$$E = \frac{p^2}{2m}$$

$$E_B = -\frac{R_y}{n^2}$$

$$E_n = -\frac{mk_e}{2}\left(\frac{q_e^2}{n\hbar}\right)^2$$

· Electron-proton energies

$$m = \frac{\sqrt{E^2 - (pc)^2}}{c^2}$$

· Invariant mass of a system

$$\Delta\lambda = \frac{h(1-\cos\theta)}{m_e c}$$

· Compton scattering

$$\lambda_{max} T = 0.2898 \cdot 10^{-2} mK$$

· Wien's Displacement Law

$$I(\lambda, T) = \frac{2\pi c k_B T}{\lambda^4}$$

· Rayleigh-Jeans Law

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda k_B T} - 1)}$$

· Planck's Law

$$E\psi(x) = \frac{-\hbar^2}{2m}\frac{\delta^2}{\delta x^2}\psi(x) + V(x) + \psi(x)$$

· Schrödinger equation

$$1 = \int_{-\infty}^{\infty} |\psi(x)|^2 \ dx$$

· Normalization condition

$$\langle a \rangle = \int_{-\infty}^{\infty} \psi^*(x)\hat{a}\psi(x) \ dx$$

h	: Planck constant, Js
q_e	: Charge of an electron C
V_s	: Stopping voltage, V
Φ	: Work function, J
$f_\circ$	: Critical frequency, Hz
p	: Momentum, $kgms^{-1}$
Δx	: Uncertainty in observed value x
E_B	: Binding energy, J
R_y	: Rydberg constant, eV
n	: Energy level, $n \in \mathbb{N}$
k_B	: Boltzmann constant
T	: Temperature, K
$\psi(x)$	: Wave function
$\psi(x)^*$	: Complex conjugate of wave function
$V(x)$	: Potential energy
$\langle a \rangle$	: Expectation value of observable a
$\hat{a}$	: Operator of observable, matrix form

Observable	Operator
$\langle x \rangle$	x
$\langle x^2 \rangle$	x^2
$\langle p \rangle$	$\frac{\delta}{\delta x}(-i)\hbar$

· *Square wells*

Infinite : $\psi''(x) = -k^2\psi(x)$

$$k^2 = \frac{2mE}{\hbar^2}$$

Finite : $\psi''(x) = \alpha^2\psi(x)$

$$\alpha^2 = \frac{2m(V-E)}{\hbar^2}$$

· *Special Relativity*

Lorentz factor : $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

time dilation	$\Delta t_\circ$ proper time	Δt observed time	$\Delta t = \Delta t_\circ \gamma$
mass expansion	$m_\circ$ proper mass	m observed mass	$m = m_\circ \gamma$
length contraction	$L_\circ$ proper length	L observed length	$L = \frac{L_\circ}{\gamma}$

7. Universal Gravitation

$$F = \frac{GMm}{r^2}$$
$$E_{p_g} = \frac{-GMm}{r}$$
$$\Delta E_{p_g} = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$
$$T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$$
$$v_c = \sqrt{\frac{GM}{r}}$$
$$v_e = \sqrt{\frac{2GM}{r}}$$
$$L = mvr$$

G : Universal gravitational constant
E_{p_g} : Gravitational potential energy
r : Distance between center of mass of two objects
T : Period of revolution, s
a : Length of semi-major axis, m
v_c : Speed in a circular orbit, ms^{-1}
v_e : Escape speed, ms^{-1}
L : Angular momentum, kgm^2s^{-1}

8. Fluid Mechanics

$$\Delta p = \rho g \Delta h$$
$$p = p_\circ + \rho g h = \frac{F}{A}$$
$$F_B = V_m \rho g$$
$$const = p + \frac{1}{2}\rho v^2 + \rho g h$$
$$F_{drag} = 6\pi\eta r v \quad \cdot \text{ laminar flow}$$
$$= \frac{1}{2}CA\rho v^2 \quad \cdot \text{ turbulent flow}$$
$$\tau = \frac{F}{A}$$
$$\eta = \frac{\tau}{{}^{\delta v}/_{\delta x}}$$
$$Re = \frac{\rho v d}{\eta}$$
$$R = A_1 v_1 = A_2 v_2$$

p : Pressure, $kgm^{-1}s^{-2}$
$p_\circ$: Pressure at surface of liquid
ρ : Density of fluid, kgm^{-3}
h : Change in height, m
A : Area, m^2
F_B : Buoyancy force, N
V_m : Volume of mass in water
η : Viscosity, $kgm^{-1}s^{-1}$
r : Radius of object experiencing drag force
C : Drag coefficient
τ : Shear stress
${}^{\delta v}/_{\delta x}$: Velocity gradient
Re : Reynolds number
d : Characteristic length, m
R : Flow rate, m^3s^{-1}

9. Constants

$$
\begin{aligned}
h &= 6.62606896 \cdot 10^{-34}\ J \cdot s \\
&= 4.13566733 \cdot 10^{-15}\ eV \cdot s \\
c &= 2.99792458 \cdot 10^{8}\ m \cdot s^{-1} \\
m_e &= 9.10938215 \cdot 10^{-31}\ kg \\
&= 0.510998910\ MeV \cdot c^{-2} \\
m_p &= 1.672621638 \cdot 10^{-27}\ kg \\
&= 938.272013\ MeV \cdot c^{-2} \\
q_e &= -1.602176487 \cdot 10^{-19}\ C \\
q_p &= 1.602176487 \cdot 10^{-19}\ C \\
k_e &= c^2 \cdot 10^{-7} \\
&= 8.9875517873 \cdot 10^{9}\ N \cdot m^2 \cdot C^{-2} \\
\mu_0 &= 4\pi \cdot 10^{-7} \\
\varepsilon_0 &= (\mu_0 \cdot c^2)^{-1} \\
&= 2.2654418729 \cdot 10^{-3} \\
R_E &= 6.371 \cdot 10^{6}\ m \\
m_E &= 5.9742 \cdot 10^{24}\ kg \\
g &= 9.80665\ ms^{-2} \\
G &= 6.67428 \cdot 10^{-11}\ m^3 \cdot kg^{-1} s^{-2} \\
R_y &= 13.6056923\ eV
\end{aligned}
$$

h : Planck constant
c : Speed of light in a vacuum
m_e : Electron mass
m_p : Proton mass
q_e : Electron charge
q_p : Proton charge
k_e : Coulomb's constant
μ_0 : Permeability constant of free space
ε_0 : Permittivity constant of free space
R_E : Earth radius
m_E : Earth mass
g : Earth surface gravity
G : Universal gravitational constant
R_y : Rydberg constant