Compact course notes

PHYSICS 334, FALL 2012

Quantum Physics 2

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0.1 Introduction

In this course, all operators will be linear.

Proposition 0.1.1. Quantum physics has 6 basic postulates that describe it completely and accurately:

1. The state of a quantum system is described by a wave function $|\psi(t)\rangle$
2. Every measurable quantity, or observable, A, is described by an operator $\hat{A}$
3. The only possible results of a measurement of A are eigenvalues of $\hat{A}$
4. The probability of measuring a specific eigenvalue a_i of an observable A is given by $|\langle A_i|\psi\rangle|^2$ for $|A_i\rangle$ the eigenvector of A with eigenvalue a_i
5. After measurement, the state is left in the eigenstate of A corresponding to the measured outcome, or $|\psi\rangle \to |A\rangle_i$
6. The time evolution of $|\psi\rangle$ is given by the Schrödinger equation

1 The Schrödinger equation

In Newtonian mechanics, given position x and velocity v at $t = 0$, all future positions can be predicted using Newton's law $F = ma$. This law in quantum mechanics is replaced by Schrödinger's equation, and initial conditions are replaced by the wave function ψ.

1.1 Definitions

- The time-dependent Schrödinger equation is given by $i\hbar\dfrac{\partial\psi}{\partial t} = \dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2} + V\psi$.

- The operator $H = \underbrace{\dfrac{-\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}}_{\text{kinetic energy}} + \underbrace{V}_{\substack{\text{potential}\\\text{energy}}}$ is termed the Hamiltonian.

- The operator $\hat{p} = \dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$ is termed the momentum operator.

- Given an observable Q with n possible outcomes Q_i, each with probability p_i of occurring, the expectation value (or weighted average) of Q is given by $\langle Q\rangle = \sum_{i=1}^{n} Q_i p_i$. If Q is continuous, $\sum \to \int$.

- The expectation value of a quantum mechanical observable is $\langle O\rangle = \displaystyle\int_{-\infty}^{\infty} \psi^*(x,t)\hat{O}\psi(x,t)dx$.

The time-independent Schrödinger equation is given by $H\psi(x) = E\psi(x)$.

The uncertainty in $\hat{O}$ is given by $\sigma_{\hat{O}} = \sqrt{\langle\hat{O}^2\rangle - \langle\hat{O}\rangle^2}$.

1.2 Ideas

- The act of normalization is setting $\displaystyle\int_{-\infty}^{\infty} |\psi(x,t)|^2 dx = 1$, which is a real-world constraint.

- Operators Q that only depend on position (eg. x^2) are such that $\hat{Q} = Q$ (eg. $\hat{x}^2 = x^2$).

- If we suppose that a solution to the Schrödinger equation is separable, i.e. $\Psi(x,t) = \psi(x)T(t)$, then $T(t) = exp\left(\frac{-iEt}{\hbar}\right)$ for $E = \frac{H\psi}{\psi}$, the energy of the system.

- Time dependence in the TISE is a result of interference.

1.3 Propositions

- Given $\psi(x,0)$we can find $\psi(x,t)$ using the time-dependent Schrödinger equation.

- The probability of finding a particle between a and b is $P_{a,b} = \int_a^b \psi^* \psi dx = \int_a^b |\psi(x,t)|^2 dx$.

- A separable state has a fixed energy (has no energy uncertainty), as $\Delta E = \sqrt{\langle H^2 \rangle - \langle H \rangle^2} = \sqrt{E^2 - E^2} = 0$.

- The most general solution to the full Schrödinger equation is a linear combination of separable solutions:

$$\Psi(x,t) = \sum_n c_n \psi_n(x) \exp\left(-iEt/\hbar\right)$$

where $c_n \in \mathbb{C}$ for all n.