Neighborhoods for classifying binary dynamics in directed networks with machine learning

LU Interdisciplinary Applied Mathematics seminar

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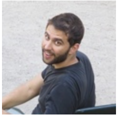

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Slides at: `jlazovskis.com/talks`

Overview

About.

- Based on work in preparation (2019-2021)
- In collaboration with the Blue Brain Project (EPFL)
- Funded by EPSRC grant "Topological Analysis of Neural Systems"

Goals.

- Neurological: Make a *in silico* model of a brain based on *in vivo* models.
- Mathematical: Distinguish neurological activity by its topological features.

Plan.

1. Neuroscience
   - Structure of the network
   - Experiments on the network
2. Mathematics
   - Topology in neuroscience
   - Neighborhoods in a digraph
3. Implementation and results
   - Classification by machine learning
   - Computational requirements

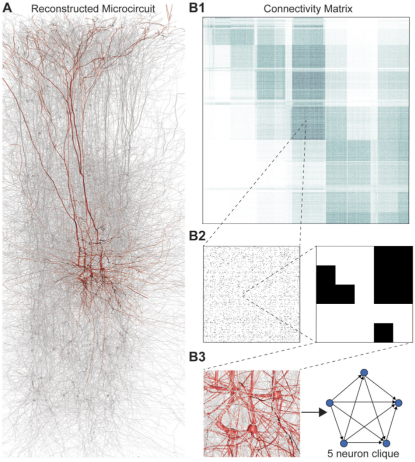
Neuroscience: Structure

Microcircuit of a rat somatosensory column, average over 6 instances:

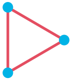
	neuroscience	*mathematics*
31 346	neurons	vertices
7 803 528	synapses	directed edges

This graph is:

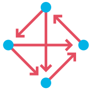
- Biologically modeled ("grown")
- Very sparse (0.8% density)
- Physically small (0.29mm^3 of brain)
- Relatively small (newest version has 4.2 million neurons and 4.8 billion synapses)
- Open source (available at `bbp.epfl.ch/nmc-portal`)
- Not a random graph (more topological features)



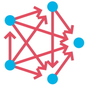
The complexity of a network can be considered via the number of *cliques*.



undirected 3-clique



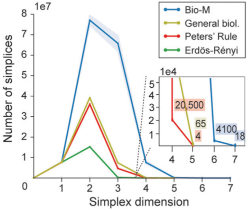
directed unordered 4-clique

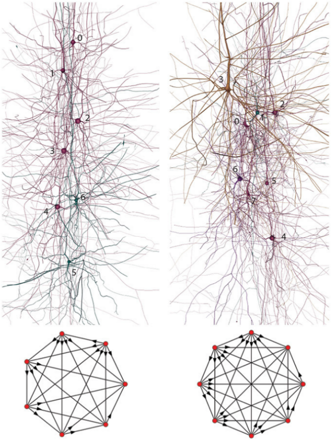


directed ordered 5-clique

Large cliques unlikely, and among those unordered cliques dominate. On n vertices:

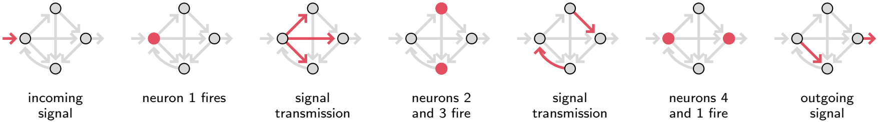
- $3^{n(n-1)/2}$ possible configurations
- $2^{n(n-1)/2}$ possible configurations as directed n-cliques
- $n!$ possible configurations as directed ordered n-cliques





Activity.

- Each neuron has an *electric potential* and a *threhsold*.
- If the potential passes the threshold, the neuron *fires* and sends its potential along outgoing synapses.
- Each synapse has a probability of signal *transmission*.
- Modern models encode *plasticity*, or the change in future transmission probability based on past transmission.

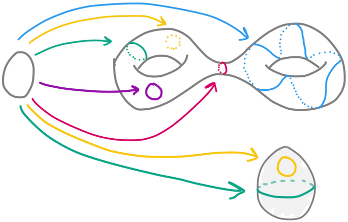


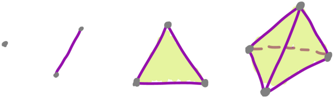
Experiments.

- External input is connected by *thalamic fibers* to receptory neurons.
- Different *stimuli* are sent to the brain (flick of a whisker)
- The propogation of activity is recorded over 250ms after the input of the signal
- 8 stimuli, 557 repetitions of each
- Data recorded as a list of values (n_i, t_i) of the neuron index and the time it fires

Mathematics: Topology

- The *topology* of a space X is reflected by classes of maps $S^n \to X$.
- The more (homotopy) classes in C_n, the more complex the space.
- The *flag complex* of a directed graph comes from associating to every directed n-clique an $(n-1)$-dimensional *simplex*.





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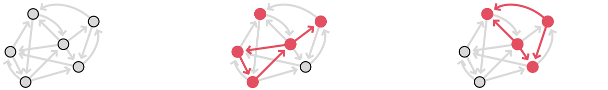
homology groups:	$H_n(X) = \ker(C_n \to C_{n-1})/\mathrm{im}(C_{n+1} \to C_n)$
Betti numbers:	$\beta_n(X) = \lvert H_n(X) \rvert$
Euler characteristic:	$\chi(X) = \beta_0 - \beta_1 + \beta_2 - \beta_3 + \cdots$
normalized Betti coefficient:	$\mathfrak{B}(X) = \dfrac{\beta_0(X)}{\#\text{ of vertices}} + \dfrac{2\beta_1(X)}{\#\text{ of edges}} + \cdots$

Mathematics: Neighborhoods in a digraph

Neuroscience uses the *firing rate* of a neuron or region for classification.
We use different parameters based on graph neighborhoods.

- Fundamental: firing rate, in degree, out degree, recioprocal connection count
- Algebraic: clustering coefficient
- Topological: Euler characteristic, Betti coefficient, density coefficient
- Spectral: adjacency, Laplacian, transition probability



structural | *active subgraph on* $[t_0, t_1]$ | *active subgraph on* $[t_1, t_2]$

In total 30 different parameters.

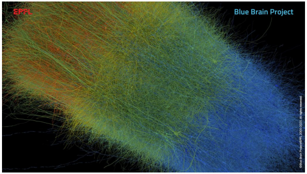
Implementation: Technical requirements

Neuroscience: EPFL

- Circuit built and experiments run on EPFL Blue Brain Project supercomputers
- 42k cores, 94TB of RAM
- Takes about 10 hours to simulate 250ms

Mathematics: University of Aberdeen

- Analysis run on UoA Maxwell HPC
- 1.2k cores, 12TB of RAM
- Takes about 2 hours to featurize each parameter, 1 minute to classify
  - Requests 40 cores, 150GB of RAM
  - Topological computations in parallel



End

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