Topological methods and soft matter physics

MMML seminar

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Jānis Lazovskis

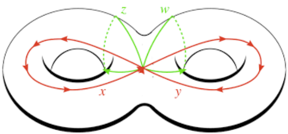
Slides at: `jlazovskis.com/talks`

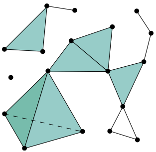
Outline.

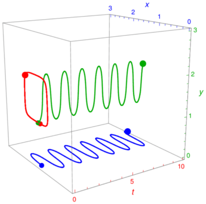
1. My background
2. Topological tools relevant to soft matter physics
3. Topics in soft matter physics amenable to topology

Keywords: Algebraic topology, homotopy theory, category theory

- X is a topological space
  - (V, S) is a *simplicial complex*
  - $\mathrm{Conf}_n(X)$ is a *configuration space*
  - The topology may depend on (Hausdorff) distance

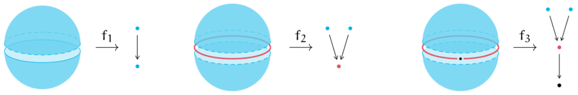


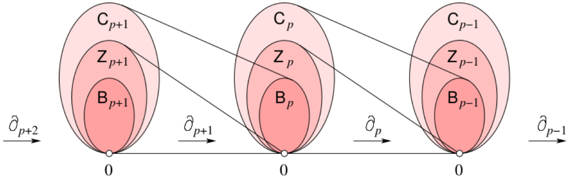




Background continued

- X may be decomposed into smaller pieces
  - The smaller pieces and how they fit together give information about X
- $\mathcal{F}$ is a sheaf over X
  - Associates simpler spaces to open sets of X
- $H_n(X; R)$ is the nth homology group of X, with coefficient in R
  - H_0 is how many connected components, H_1 is how many "holes" there are
  - When $R = \mathbf{R}$, the *Betti number* $b_n = |H_n|$ is a convenient and simple descriptor





Topology and graphs for biology: Structure

Joint project “Topological Analysis of Neural Systems” with EPFL.

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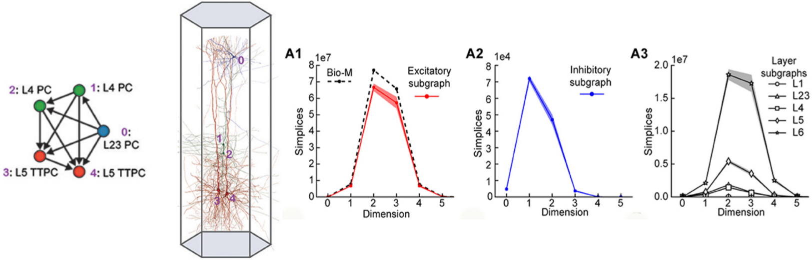
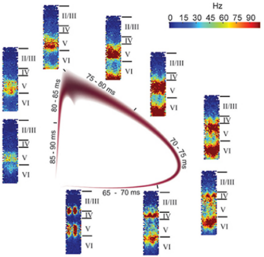
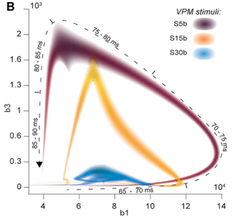
- Build a biologically realistic model of a brain in a computer
- Feed it biologically realistic singals
- Analyze neural activity with topology



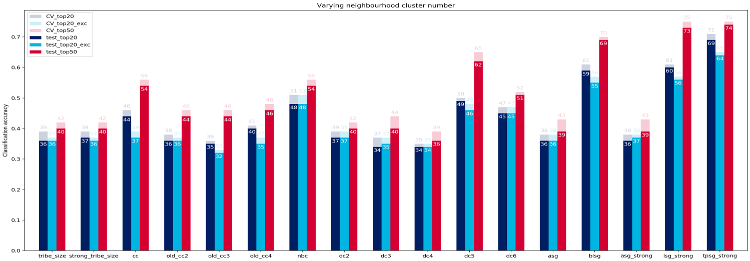
Topology and graphs for biology: Activity

Setting: Stimulate neurons with electricity at $t = 0$, record activity until $t = 250ms$.

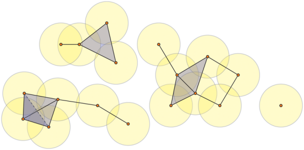




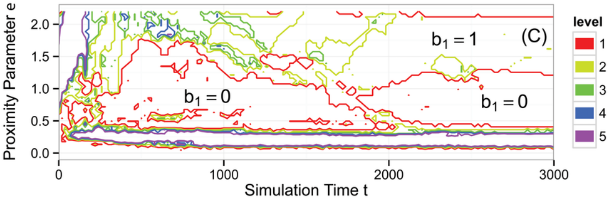
Observation: Topological classification works with a small number neighborhoods.



Key ideas: Vietoris–Rips complex, persistent homology



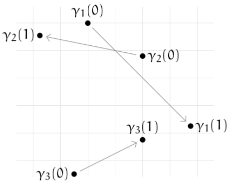
- Build (a) topological space(s) from input data
- Determine topological properties of the space
- Interpret the properties in the context of the original data

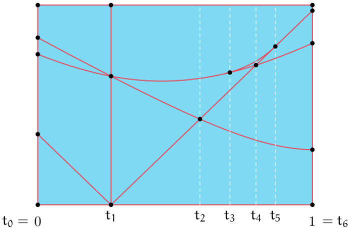


Tools: $\mathrm{Conf}_n(X) = \{S \subseteq X \ : \ |S| = n\}$ and $\mathrm{Ran}_{\leqslant n}(X) = \{S \subseteq X \ : \ 0 < |S| \leqslant n\}$

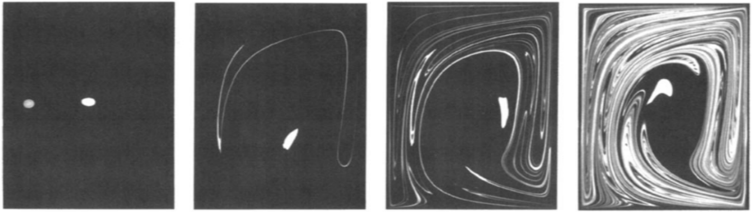
Paths: A *path* is a continuous function $\gamma \colon [0,1] \to \mathrm{Conf}_n(X)$

- How many paths are there between two chosen points?
- Which is the shortest path? Is it unique?
- If γ is a loop, that is, $\gamma(0) = \gamma(1)$, is it contractible?

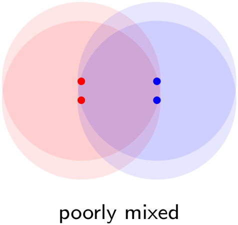




Goal: Determine if two fluids have "mixed well".

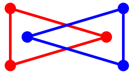


- Identify n points in both fluids as $P_1(t=0)$ and $P_2(t=0)$
- Compute the Hausdorff distance between $P_1(t \gg 0)$ and $P_2(t \gg 0)$



Opposite approach: Keep track of how topological properties of $\mathrm{VR}(P_i(t))$ change.

poorly mixed



well mixed

Thank you for your attention.

References.

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