# Continuous and discrete dynamic topology

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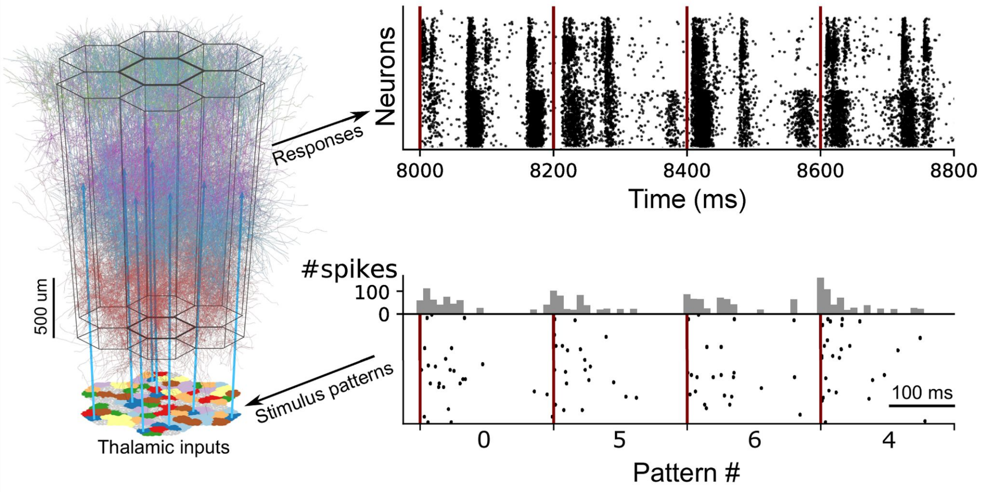
# Discrete dynamics in topology

*How can topology help local binary dynamics classify global dynamics?*




*University of Aberdeen, Neuro-Topology group*

- Ran Levi
- Jason Smith
- Henri Riihimäki
- Dejan Govc
- Pedro Rodrigues da Conceição
- Dejan Govc

*EPFL, Blue Brain Project*

- Kathryn Hess
- Daniela Egas Santander
- Michael Reimann
- Matteo Santoro
- Andras Ecker
- Vishal Sood
- Sirio Bolanos-Pouchet
- Nicolas Ninin
- …

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EPFL

# Structure

**BlueBrain V5 connectome**

**Layer structure**

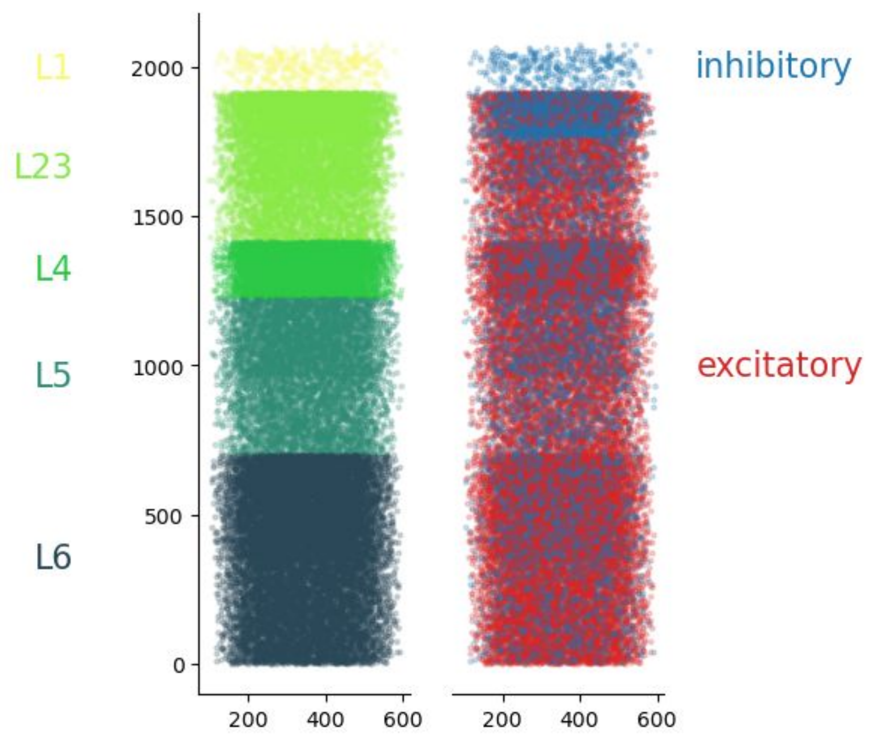
- Higher (L1) = inhibition
- Lower (L6) = information processing

**Neuron characteristics**

- 31346 in total
- 55 electro-morphological classes

**Other facts**

- Diameter is 4
- High dimensional simplices are over-represented
- Reciprocal connections preferentially appear in high-dimensional simplices




Markram et al, 2015. **Reconstruction and simulation of neocortical microcircuitry**

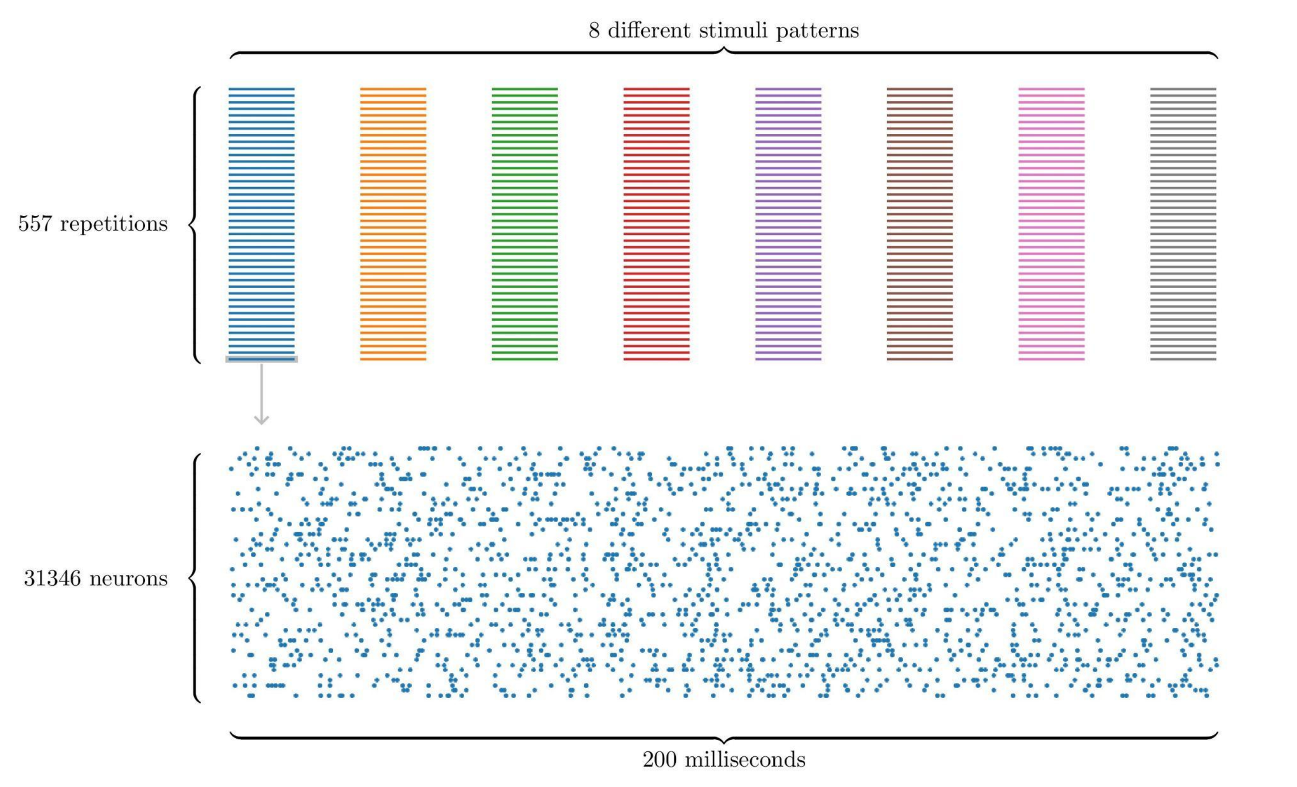
# Activity

**Stimulated activity**

Reconstruction is stimulated from an “at rest” state.

Transmission is probabilistic, response can not be predicted

Every spike index and time is recorded




Riemann et al, 2022. **Topology of synaptic connectivity constrains neuronal stimulus representation**

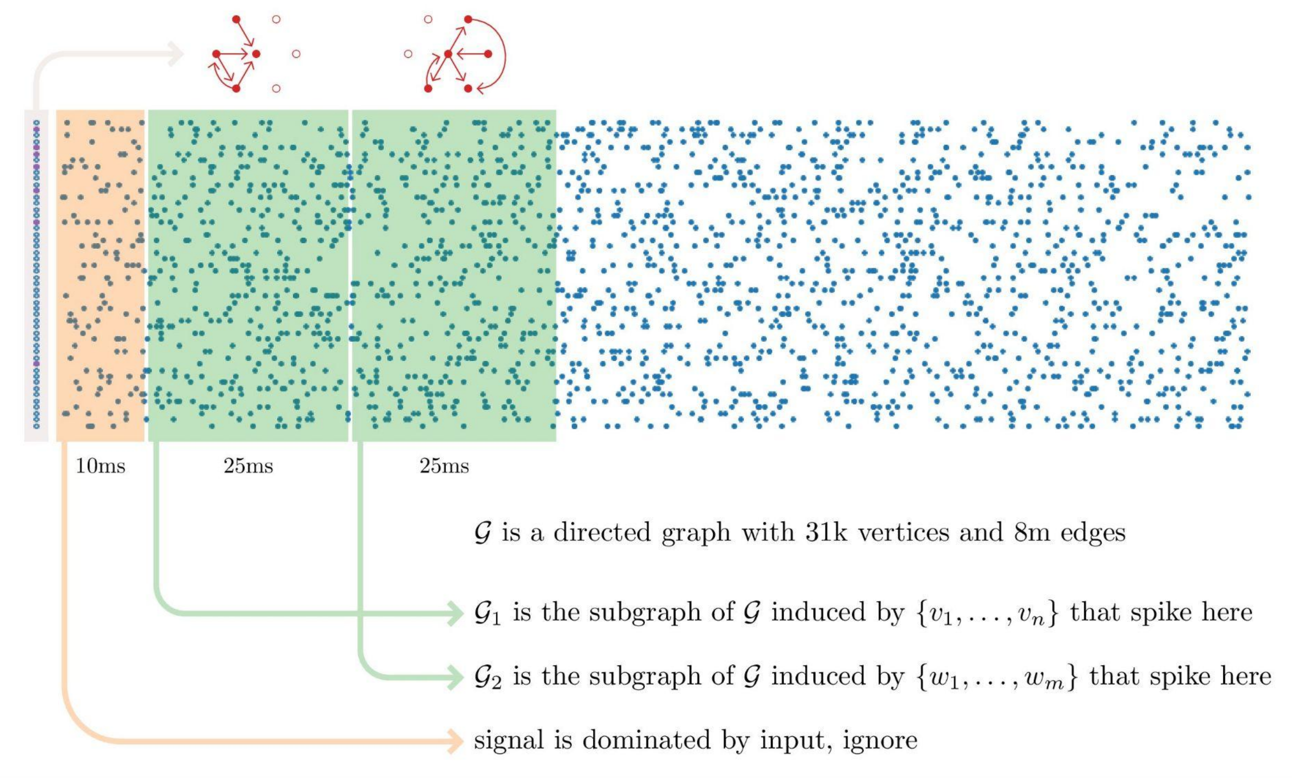
# Activity extraction

**Extracting features**

**1.** Consider two "active subgraphs" of the full graph for each repetition

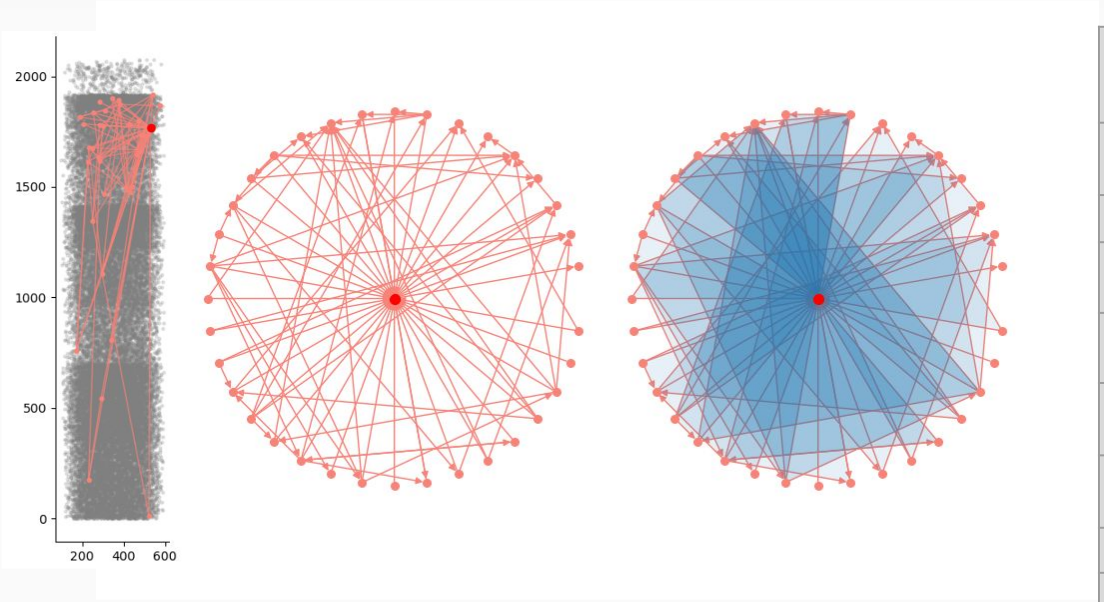
**2.** Consider two "active neighbourhoods" of *N* vertices in the active subgraph

**3.** Assign a numerical value to every active neighbourhood




Conceição et al, 2022. **An application of neighbourhoods in digraphs**

# Parameters




| | | |
|---|---|---|
| clustering coefficient | *Fagiolo (2007) generalizing Watts–Strogatz (1998) to digraphs* | 0.043 |
| transitive clustering coefficient | *ratio of all 3-cliques at v to all possible 3-cliques at v* | 0.051 |
| neighbourhood size | *size of closed neighbourhood* | 36 |
| number of reciprocal connections | *add 1 if u→v and v→u both exist* | 1 |
| adjacency spectral gap | *eigenvalues of adjacency matrix* | 1 |
| Chung Laplacian spectral gap | *of largest strongly connected component* | 0.5 |
| transition probability spectral gap | *eigenvalues of transition probability matrix* | 0.707 |
| Euler characteristic | *alternating sum of Betti numbers* | 1 |
| normalized Betti coefficient | *weighted sum of Betti numbers, weighted by dimension d and number of d-simplices* | 0.027 |
| density coefficients | *ratio of (d+1)-cliques to d-cliques, normalised to be 1 on complete graphs* | {0.028, 0.02, 0, 0} |

Used to:

- select *N=50* neighbourhoods
- assign numerical values to active neighbourhoods

Lütgehetmann D et al, 2020. **Computing Persistent Homology of Directed Flag Complexes**.

*** terminology**

# Overview: the "pipeline"

***1. Select** neighbourhoods:*

Compute graph / topological parameters for all neighbourhoods, select top *N=50* by parameter $P_1$ value

***2. Measure** active neighbourhoods:*

For each selected neighbourhood, compute parameter $P_2$ value on each of *B=2* active subgraphs

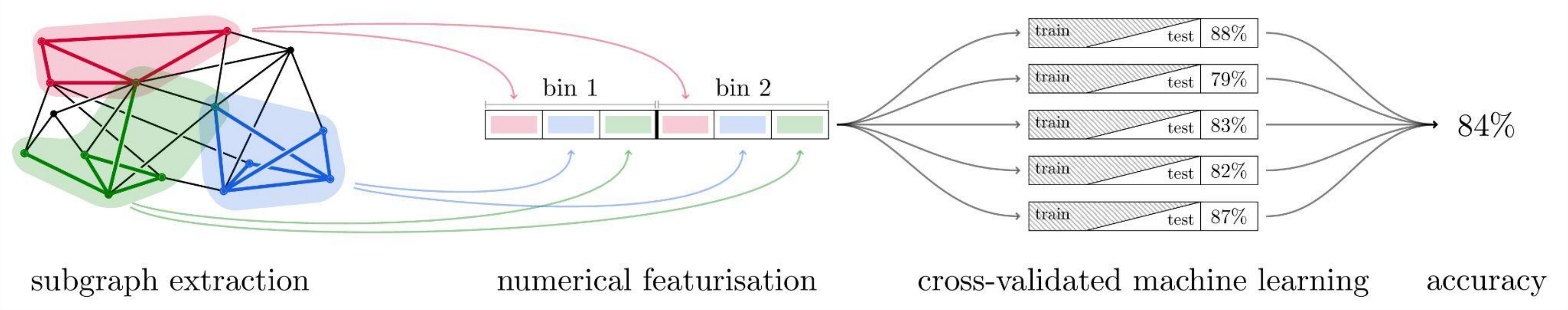
***3. Classify** with measurements as feature vectors*

Length of feature vector is $N \cdot B$, number of feature vectors is *(8 signals)·(557 repetitions)*

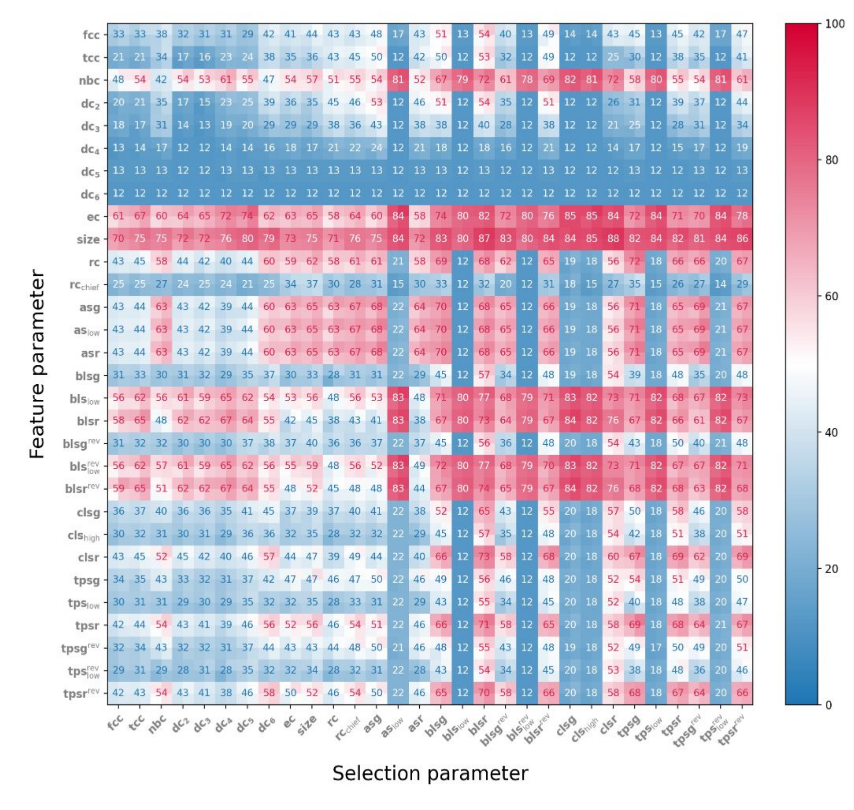
Classify with support vector machines (SVM) with 60/40 train/test five different ways

***4. Validate** with baseline results*

Random measurements, random neighbourhood choices, random ambient graph



# Results and extensions




*Classification accuracy of ~88% when:*

- selecting by a **spectral** parameter
- featurising by neighbourhood **size**

*Observations:*

- Active neighbourhood size is firing rate
- Euler characteristic is a good classifier
- Density coefficients not useful

*Development:*

- github repo for easy parameter measurement
- Double selection for better results

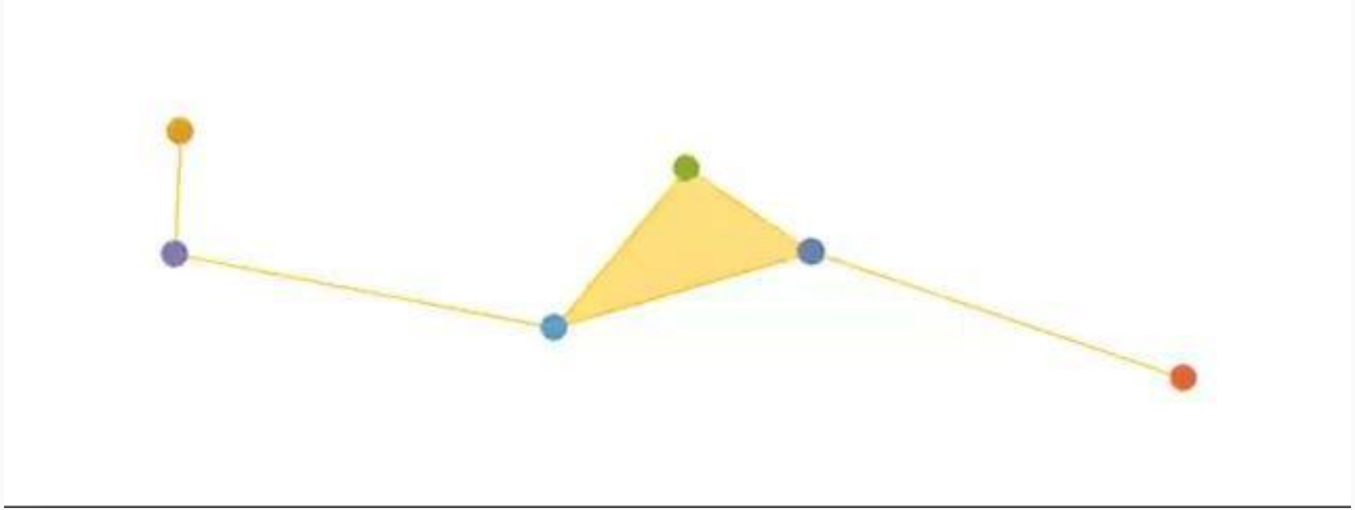
Santander et al, 2023. ***github.com/danielaegassan/connectome_analysis***

# Continuous dynamics in topology

*How can spatial change be quantified as topological change?*

**Assumptions:**

- Euclidean space
- Linear paths
- Unique locations
- Times with pairwise equal distance are measure zero



Barbara Giunti, *TU Graz*
David Millman, *Montana State University*

# Zigzags of simplicial maps

**Movement as algebra**

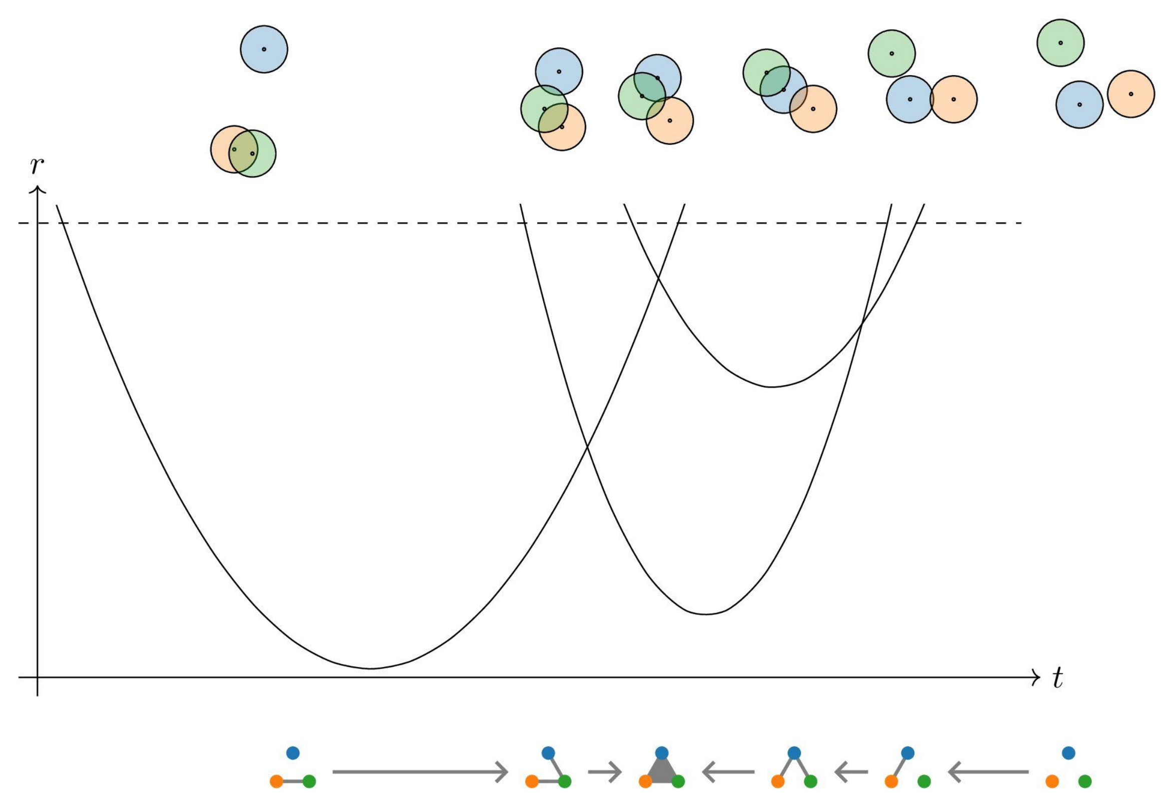
- Fix a radius
- Every pair of points defines a quadratic distance function

*Entering a parabola (left):*
- adds an edge

*Exiting a parabola (right):*
- removes an edge

**Motivation**

Recover later homology from earlier homology, without recomputing everything



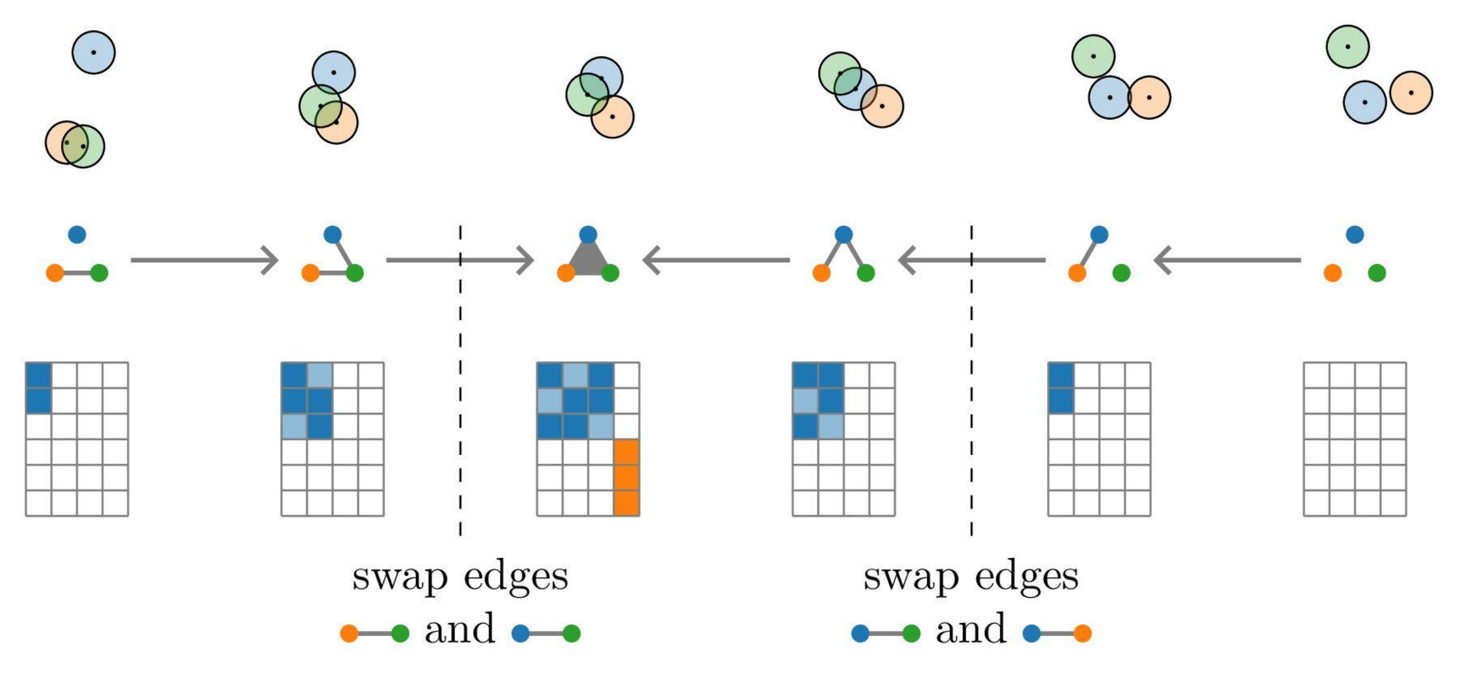


Carlsson G, de Silva V (2010). **Zigzag persistence**
Cohen-Steiner et al (2006). **Vines and vineyards**

# The (un)reduced boundary matrix

The matrix *B* has one row for each simplex, arranged in increasing dimension and increasing entrance time:

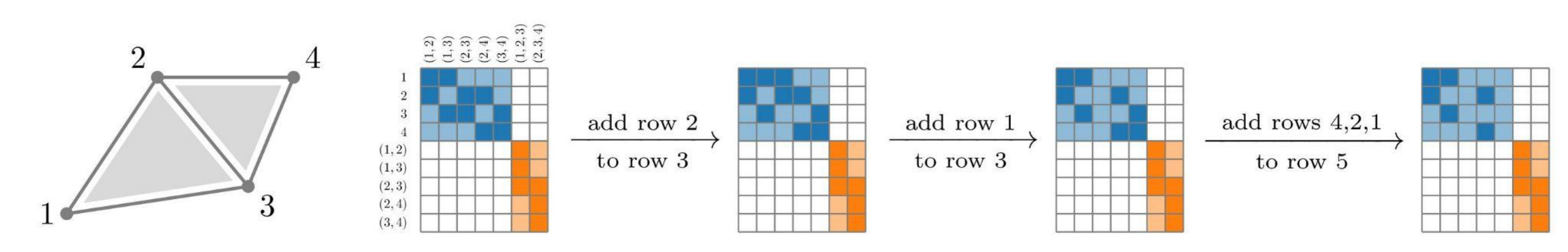
$$B_{i,j} = \begin{cases} 1 & \text{simplex of row } i \text{ is a face of simplex of column } j \\ 0 & \text{else} \end{cases}$$




* terminology

# The reduction algorithm

Add earlier columns (in modulo 2) to later columns to ensure one pivot per row:




Gives a decomposition $R = BU$

- $B$ is the (unreduced) boundary matrix
- $U$ is an upper triangular matrix recording the column operations
- $R$ is the reduced boundary matrix

**Approach:** Add / remove columns from $R$, use $U$ to ensure removal keeps $R$ reduced

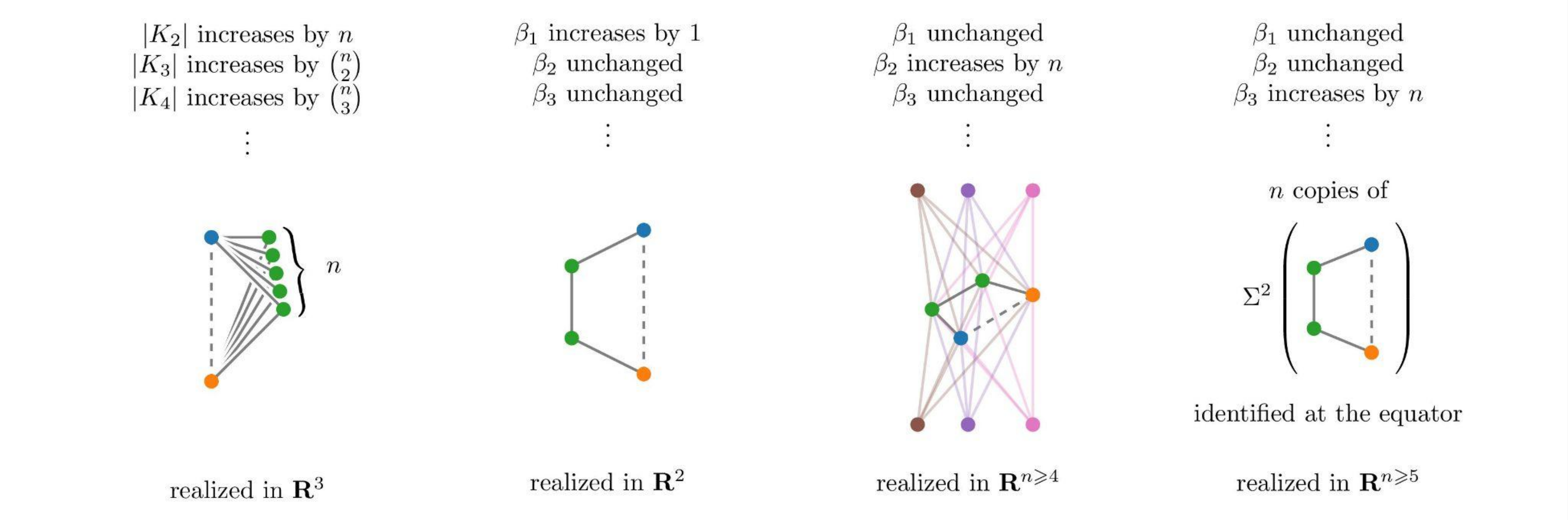
Edelsbrunner H, Harer J, 2010. **Computational Topology: An Introduction.**

* terminology

# **Adding** a new edge

Adding a single edge can may have different consequences for:

- number of n-simplices
- number of classes in $H_n$



# **Removing** an existing edge

Removing a **positive** edge

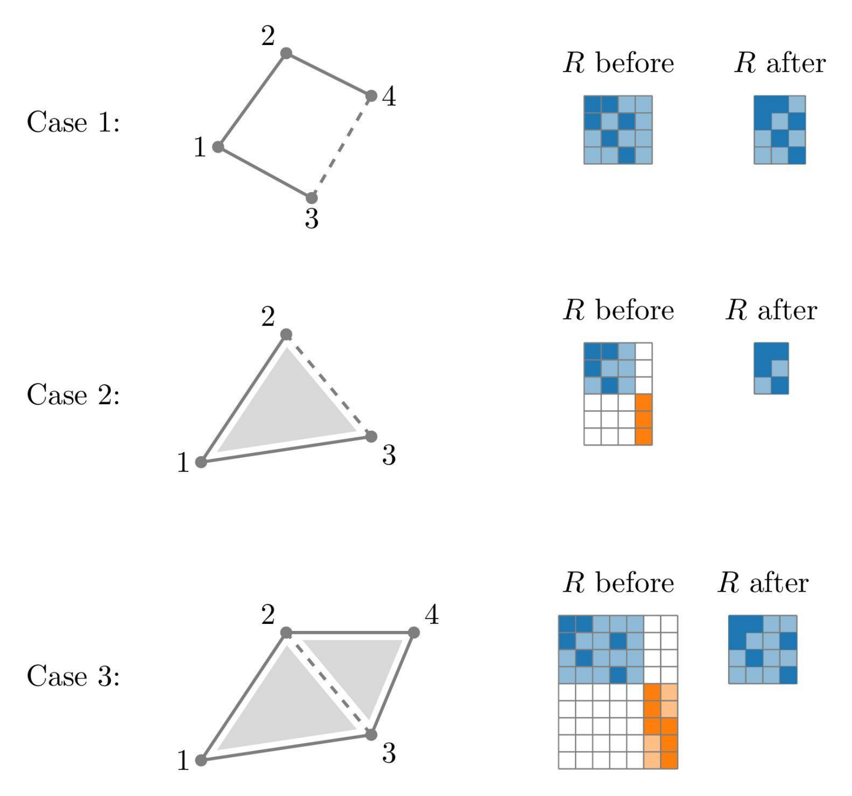
- its column is 0
- pivots in *R* won't change if this column is removed

Remove

- Row and column of edge
- Rows and columns of all cofaces

Clear:

- ***R* after is still reduced**



# **Removing** an existing edge

Removing a **negative** edge

- its column is not 0
- pivots in *R* **may** change if this column is removed

Remove

- Row and column of edge
- Rows and columns of all cofaces

Almost clear:

- ***R* after is still reduced**

Carlsson G, de Silva V (2010). ***Zigzag persistence.*** Foundations of Computational mathematics. 10, 367–405. doi: https://doi.org/10.1007/s10208-010-9066-0

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Santander DE et al (2023). ***General functions to analyze connectomes.*** https://github.com/danielaegassan/connectome_analysis