ESP Math 182 | # Worksheet 19 | **Spring 2018**

20 March 2018

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1. **Warm up:** Let $f(x) = \begin{cases} e^{-1/x^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$

   (a) Find $f'(0)$, $f''(0)$, and $f'''(0)$.

   (b) For what values of x is $f(x) = 0$?

   (c) Make a guess as to what is the Taylor series of f at 0.

2. Consider the series $f(x) = \sum_{n=0}^{\infty} \frac{(2x+3)^k}{k!}$.

   (a) Find the derivative $f'(x)$ of the series and rewrite it in terms of $f(x)$.

   (b) Using part (a), give the nth derivative of $f(x)$. Do not simply keep taking derivatives of the series.

   (c) What common function is $f(x)$ equal to?

3. Let $f(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{2^k k!}$.

   (a) Show that $f(x)$ has infinite radius of convergence.

   (b) Show that $f'(x) = xf(x)$.

4. Recall that $f^{(k)}(x)$ means the kth derivative of f.

(a) Complete the following expressions:

$$\sin^{(k)}(x) = \begin{cases} \rule{2cm}{0.4pt} & \text{if } k = 4n, \\ \rule{2cm}{0.4pt} & \text{if } k = 4n+1, \\ \rule{2cm}{0.4pt} & \text{if } k = 4n+2, \\ \rule{2cm}{0.4pt} & \text{if } k = 4n+3, \end{cases} \qquad \cos^{(k)}(x) = \begin{cases} \rule{2cm}{0.4pt} & \text{if } k = 4n, \\ \rule{2cm}{0.4pt} & \text{if } k = 4n+1, \\ \rule{2cm}{0.4pt} & \text{if } k = 4n+2, \\ \rule{2cm}{0.4pt} & \text{if } k = 4n+3, \end{cases}$$

where n is any integer.

(b) Use part (a) to give the Taylor series for $\sin(x)$ and $\cos(x)$ at $x = 0$.

(c) Use part (b) to decide if the following series converges, and if yes, to what:

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1}(2n+1)!}$$

5. Find power series representations for the following functions.

(a) $\dfrac{1}{1-x}$

(b) $\dfrac{1}{1-x^2}$

(c) $\dfrac{x}{1-x}$

(d) $\dfrac{x^2}{1-x^3}$