Topological methods for computational ecology

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Līdzfinansē Eiropas Savienība · Nacionālais attīstības plāns 2027 · Latvijas Zinātnes padome · Latvijas Universitāte · PostDoc Latvia

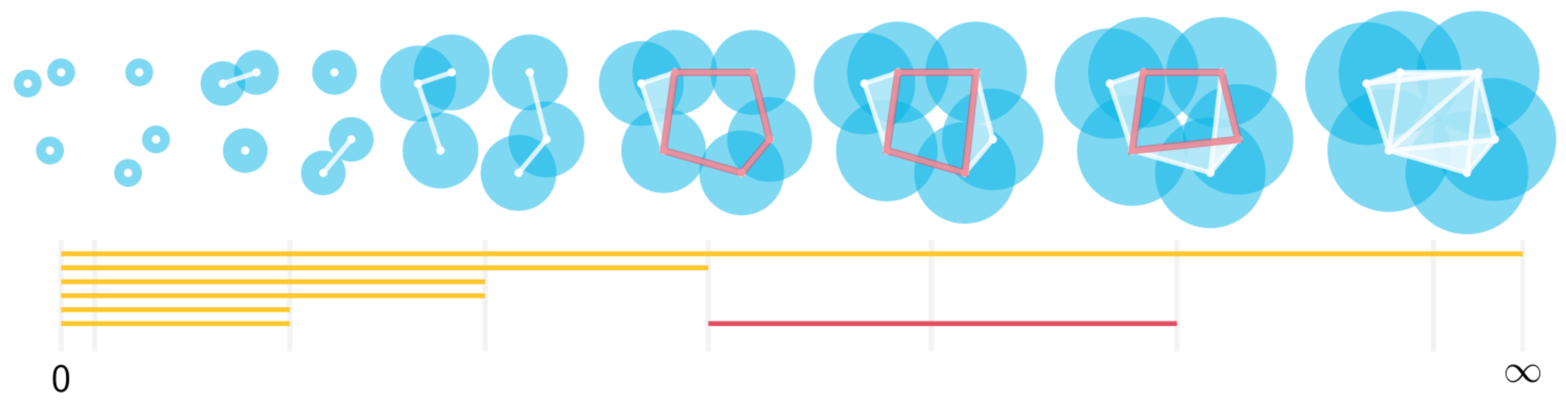
Funding for JL partly provided by the Latvian Council of Science, 1.1.1.9 Research application No 1.1.1.9/LZP/1/24/125 of the Activity "Post-doctoral Research" "Efficient topological signatures for representation learning in medical imaging"

Interdisciplinary Institute · University of Aberdeen 1495

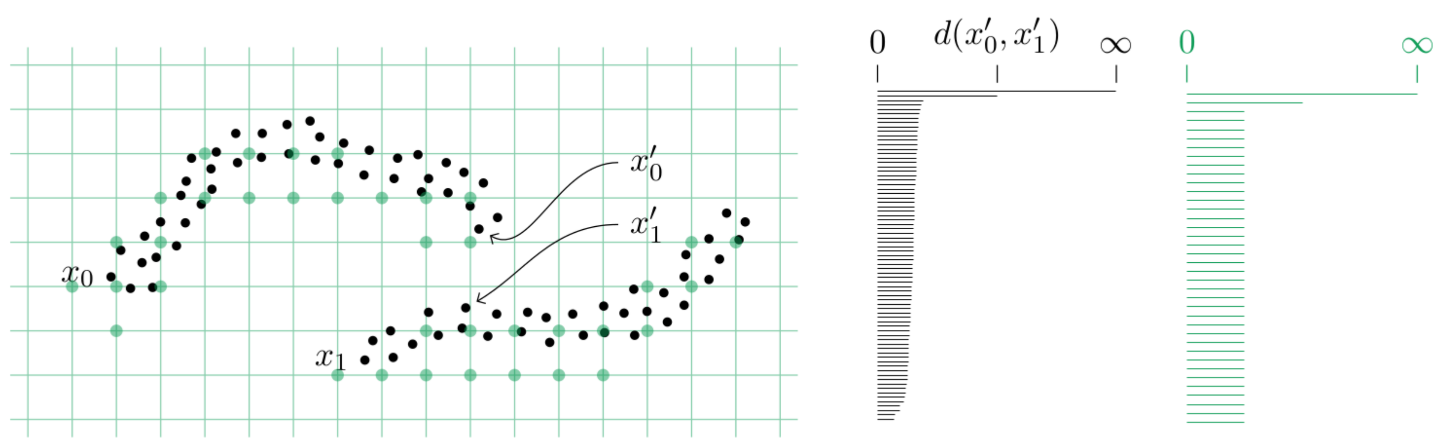
Interdisciplinary Research and Education Symposium, October 8, 2025
"Environment & Biodiversity" and "Data & Artificial Intelligence"

Topological data analysis

TDA provides a way to infer fundamental information from a limited sample. One of the main tools in TDA is **persistent homology** (PH), which describes fundamental topological information from a point cloud in a quantifiable and comparable way, using PH diagrams in topological degrees $0, 1, 2, \ldots$



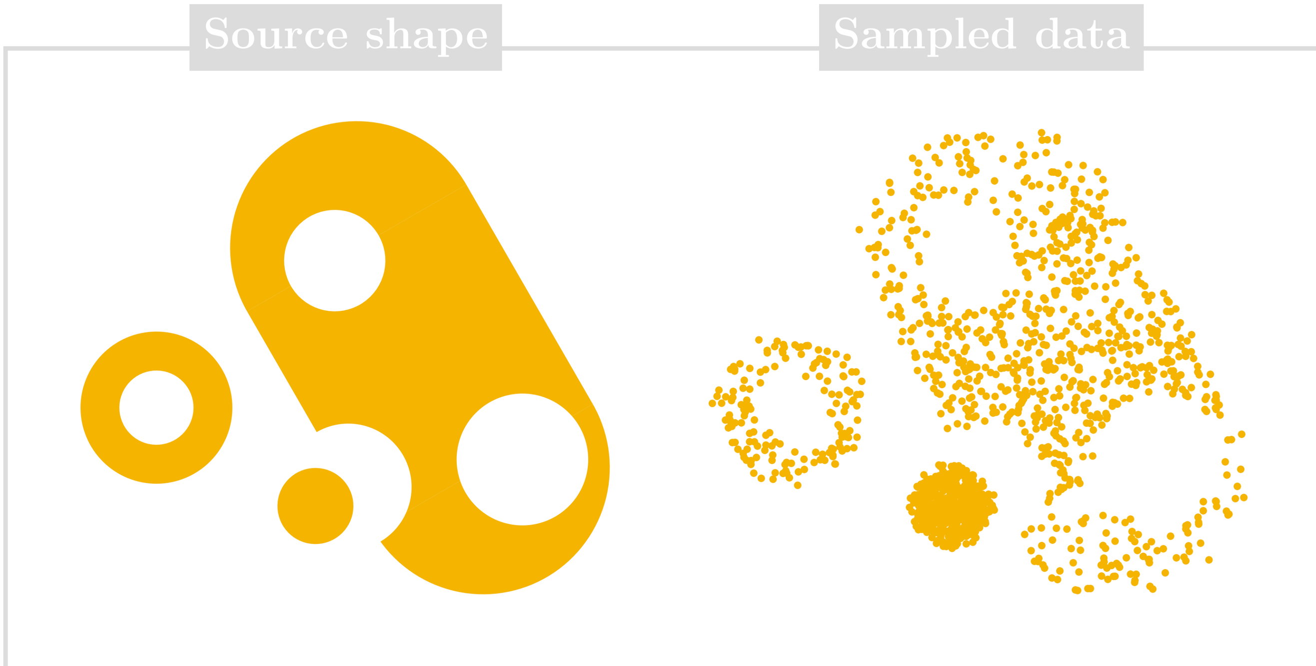
The smaller the difference $d(D(X), D(Y))$ between two PH diagrams $D(X), D(Y)$, the more certain we can be that there is a small topological difference between the underlying point clouds X, Y. We are interested in modifying datasets by *barycentric subdivision* (adding points), *sparsification* (removing points), and *aligning to a grid* (discretizing points).

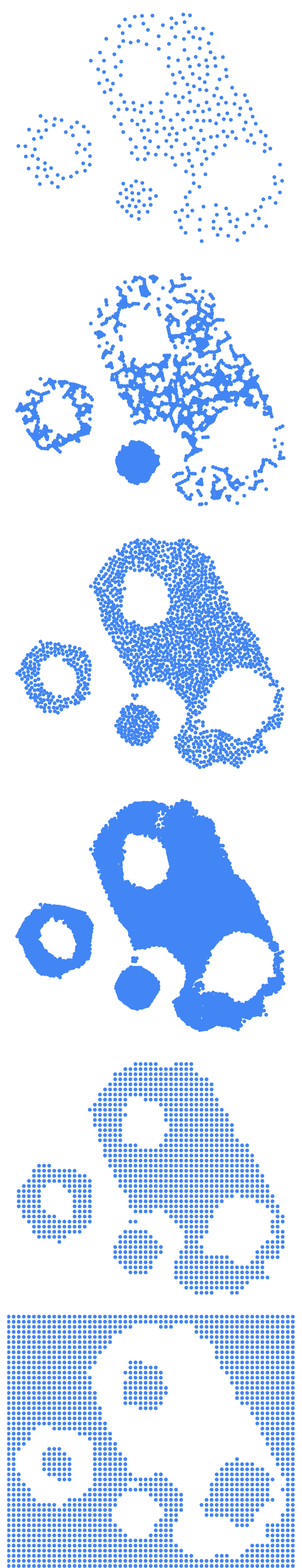


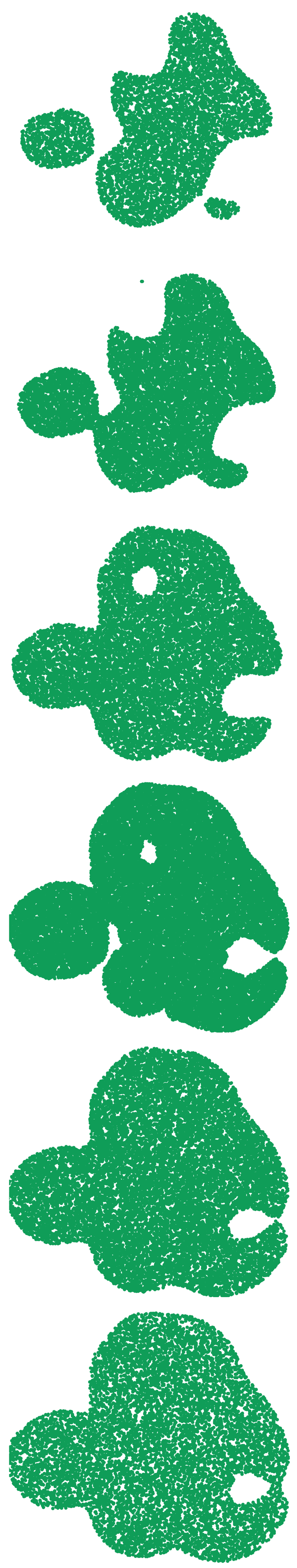
Let $X = \{x_1, x_2, \ldots, x_n\}$ be a point cloud in N-dimensional space. This set becomes $B(X, r_b)$ after adding points at a distance r_b, or $S(X, r_s)$ after removing points at a distance r_s, or $G(X, r_g)$ after discretizing the points at a step size r_g. We prove that the PH diagram in degree 0 after nested applications of these construtions is within a precise distance of the original diagram:

$$d(D(X), D(G(S(B(X, r_b), r_s), r_g))) \leq \frac{r_b}{4} + \frac{r_s}{2} + \frac{\sqrt{N} r_g}{2}.$$

Interdisciplinarity and novelty: We use computational topology to define spaces that are provably similar to niche hypervolumes, and describe their topological properties in terms of features of hypervolumes. Knowing the volume, connectedness, convexity of the hypervolume is of interest to ecologists, and we demonstrate a new way to think about and compute these values.





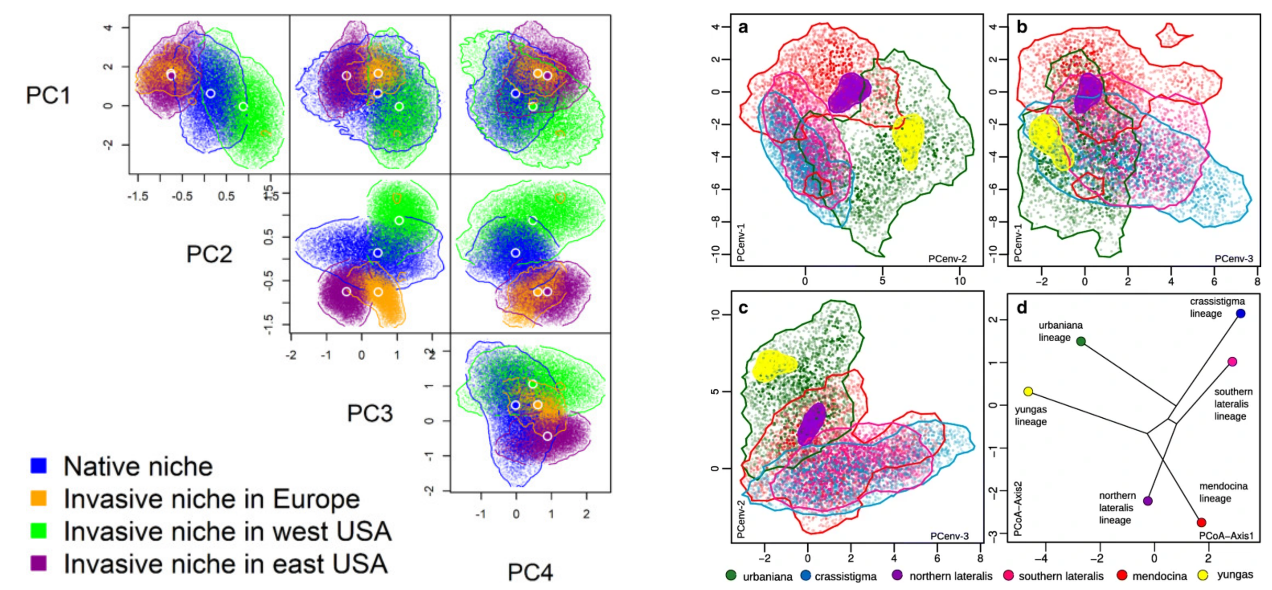


Images center left: Our methods applied to the input data set: barycentric subdivision at a maximum radius r_b to add new points, sparsification at a minimum radius r_s to remove points, aligning to a grid with interval size r_g to be able to take the complement. Used values (top to bottom) are $r_b = .1, .1, .3, .3, .3, .3$ and $r_s = .1, .02, .05, .01, 0, 0$ and $r_g = 0.8$.

Images center right: The method of Blonder et al. applied to the input data set: requested quantile percent rq to determine the density of points, standard deviation sd count to determine the spread of points. Used values (top to bottom) are $rq = 75\%, 80\%, 90\%, 98\%, 95\%, 95\%$ and $sd = 4, 2, 3, 1, 3, 5$.

Functional ecology

Understanding how species interact with each other and their environment is central to ecology. The biotic and abiotic factors in which the species can exist is its **niche**, and as a mathematical object, its **hypervolume**. This concept is widely used because of its intuitive abstract definition and the abundant availability of data, in particular public databases.



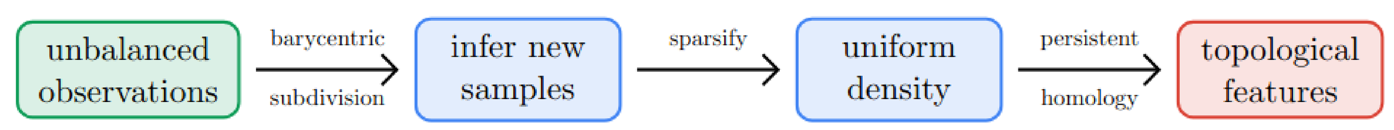
Left: Zhang et al. "To invade or not to invade? Exploring the niche-based processes underlying the failure of a biological invasion using the invasive Chinese mitten crab", *Science of the Total Environment*, 2020

Right: Salariato et al. "Ecological and spatial patterns associated with diversification of South American Physaria (Brassicaceae) through the general concept of species", *Organisms Diversity & Evolution*, 2021

Analyzing hypervolumes helps forecast species distribution, extinction risks, and responses to a changing climate. The current state of the art in computing the niche from limited observations uses **kernel density estimators** (KDE), developed by Blonder et al ("New approaches for delineating n-dimensional hypervolumes", *Methods Ecol Evol*, 2018).

- **KDE**: *infers many new samples from one; blurs sparse observations*
- **PH**: *infers one new sample from many; controls topological changes*

The KDE approach is statistical and provides no guarantee against distorting the topological features of the input. Such an approach also reinforces convexity assumptions. We reconsider how hypervolumes are constructed, approaching them with topology.



State of the project: One publication exploring this connection, another nearing submission to make precise the mathematics, and code to implement our methods, using the **GUDHI** mathematical library (`gudhi.inria.fr`). We will follow this with publications for ecological audiences, to compare, apply, and interpret our methods on datasets.

'Holey' niche!
Finding holes in niche hypervolumes using persistence homology

Conceição, Morimoto: *J.Math.Biol.*, 2022

TopoAware:
Topologically aware constructions for ecological hypervolumes

Lazovskis: *GitHub*, 2025